# LAB 6: Rotational Inertia of Toilet Paper

**Objective:**

To identify and utilize the relationship between linear acceleration, angular acceleration, and rotational inertia. As well as become familiar with the overall concept of rotational inertia.

**Theory:**

If we apply a single unbalanced force, F, to an object, the object will undergo a linear acceleration, a, which is determined by the unbalanced force acting on the object and the mass of the object. The mass is a measure of an object's inertia, or its resistance to being accelerated. Newton's Second Law expresses this relationship:

$$F = ma \qquad \text{(Equation 1.0)}$$

If we consider rotational motion, we find that a single unbalanced torque, $\tau$, produces an angular acceleration, $\alpha$, which depends not only on the mass of the object but on how that mass is distributed. The equation which is analogous to F = ma for an object that is rotationally accelerating is:

$$\tau = I\alpha \qquad \text{(Equation 1.1)}$$

Where the Greek letter tau ($\tau$) represents the torque in Newton-meters, $\alpha$ is the angular acceleration in radians/sec$^2$, and I is the moment of inertia in kg-m$^2$. The moment of inertia is a measure of the way the mass is distributed on the object and determines its *resistance* to angular acceleration. Every rigid object has a definite moment of inertia about any particular axis of rotation. Here are a couple of examples of the expression for I for two special objects:

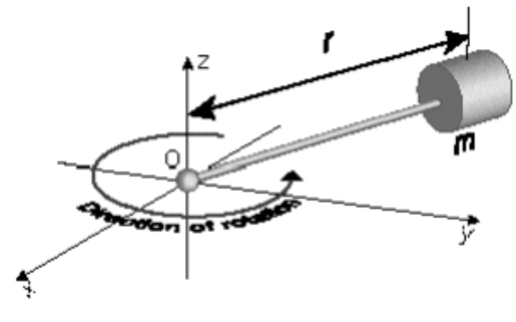


Figure 7.1: One point mass m on a weightless rod of radius $r$ ($I = mr^2$).

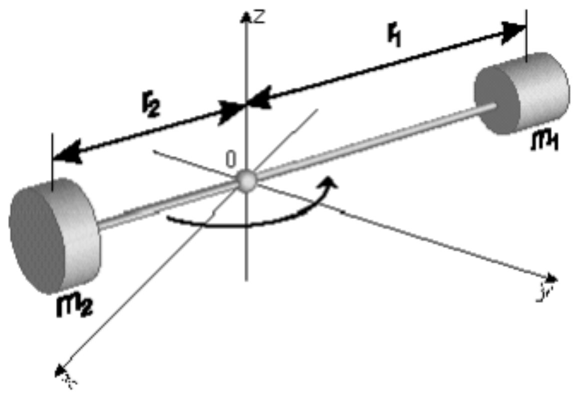


Figure 7.2: Two point masses on a weightless rod ($I = m_1r^2 + m_2r^2$).

To illustrate we will calculate the moment of inertia for a mass of 2 kg at the end of a massless rod that is 2 m in length (Fig. 7.1 above):

$$I = mr^2 = (2\text{ kg})(2\text{ m})^2 = 8\text{ kg m}^2$$

If a force of 5 N were applied to the mass perpendicular to the rod (to make the lever arm equal to r) the torque is given by:

$$\tau = Fr = (5\text{ N})(2\text{ m}) = 10\text{ N m}$$

Furthermore, recall that linear or *tangential* acceleration can be found using the angular acceleration multiplied by the radius of rotation, shown below:

$$a_t = \alpha r \qquad \text{(Equation 1.2)}$$

* Hint: think back kinematics of free-fall to successfully complete this lab*

**Equipment:**

Two rolls of standard 2-ply toilet paper, high-speed video recording device, two meter sticks.

**Procedure:**

***DO NOT ATTEMPT TO DROP TP ROLLS FROM HEIGHTS AT THIS TIME***

Within your group, work out the theoretical concepts and derive the necessary equations to identify what the ratio of the two heights of the different Toilet Paper rolls need to be in order to reach the ground at the EXACT same time. One of the toilet paper rolls will be held by the paper and unrolled down to the ground. The second toilet paper roll will be dropped, into free-fall, at the exact same time as the other roll.

Next, confirm calculations and thoroughly explain reasoning behind your calculation of the ratio of the two heights to your teacher when you are ready.

Last, make the drop! If the two rolls of toilet paper, one which is unrolling and one with is in free-fall, reach the ground at the exact same time on the first attempt then extra-credit will be applied to your lab write-up. If they do not reach the ground at the same time, perform a qualitative error analysis and then contact the teacher when you are ready for your second attempt. Only a maximum of three attempts can be made for this experiment.

**Questions:**

What was your final ratio of the two heights?

If you were to drop paper towel rolls, how would that affect your rotational inertia?

Perform an error analysis for your measurements and determine your standard of deviation for your ratio. Was this deviation represented in your final experimentation when you tested your calculations? Why or why not?

To begin the exercise, we set our variables (H=height for dropped roll, h=height for unrolled roll, r = inner radius, R = outer radius), then identified the time it takes for the dropped roll to hit the ground using standard kinematics:

$$t_{drop} = \sqrt{\frac{2H}{g}}$$

Next, we did the same thing for the unrolling toilet paper roll:

$$t_{unroll} = \sqrt{\frac{2h}{a}}$$

Of course, if we want them to hit at the same time, the times must be equal, therefore we can show:

$$\frac{H}{h} = \frac{g}{a}$$

Obviously, what we really need to focus our efforts on is finding the linear acceleration of the unrolling roll. To save ourselves some time, we started by looking up the moment of inertia for a cylinder:

$$I = \frac{1}{2}M(r^2 + R^2)$$

Using the parallel-axis theorem to account for the unrolled roll rotating about its outer radius we find:

$$I = \frac{1}{2}M(r^2 + R^2) + MR^2 = \frac{1}{2}M(r^2 + 3R^2)$$

Next, we can use a free body diagram to identify the net torque on the roll as MgR, and use Newton's 2nd Law for Rotational Motion to find the angular acceleration:

Torque = I x Alpha, solve for alpha.

Since linear acceleration can be found from angular acceleration multiplied by the radius of rotation (R):

$$a = \alpha R = \frac{2gR^2}{r^2+3R^2}$$

Finally, since we're looking for the ratio of the dropped height to the unrolled height:

For a toilet paper roll of inner radius .009m and outer radius R=.0385m (our school rolls from the janitor supply closet):

Reference:

Ehrlich, R. (1997). *Why toast lands jelly-side down: Zen and the art of physics demonstrations*. Princeton, NJ: Princeton University Press.